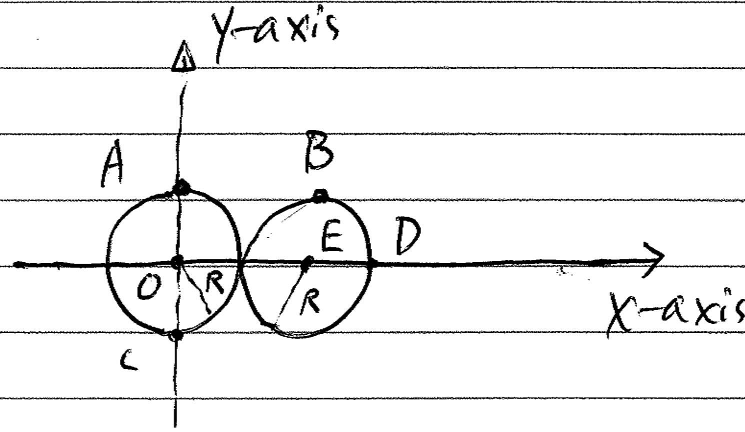
Solutions to Physics 9C-A Final (2013)

1-(a) Referenced to infinity,



$$V_A = V_A(Q) + V_A(-Q)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

$$+ \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-Q)}{(R^2+(2R)^2)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\left[1-\frac{1}{\sqrt{5}}\right]$$

$$V_B = V_B(Q) + V_B(-Q)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{(R^2+(2R)^2)^{1/2}} + \frac{(-Q)}{4\pi\varepsilon_0 \cdot R}$$

$$= \frac{(-Q)}{4\pi\varepsilon_0}\frac{1}{R}\left[1-\frac{1}{\sqrt{5}}\right]$$

$$\therefore V_A - V_B = \frac{1}{2\pi\varepsilon_0}\frac{Q}{R}\left[1-\frac{1}{\sqrt{5}}\right]$$

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1-(b)

$$\vec{E}_c = \vec{E}_c(Q) + \vec{E}_c(-Q)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}(-\hat{j}) + \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-Q)}{(R^2+(2R)^2)^{3/2}}$$

$$\cdot\left((-2R)\hat{i} + (-R)\cdot\hat{j}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\left[\frac{2}{5\sqrt{5}}\hat{i}\right] + \frac{Q}{4\pi\varepsilon_0}\frac{1}{R^2}\left(\frac{1}{5\sqrt{5}} - 1\right)\hat{j}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\left[\frac{2}{5\sqrt{5}}\hat{i} + \left(1 - \frac{1}{5\sqrt{5}}\right)(-\hat{j})\right]$$

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1-(c)

$$\vec{E}_D = \vec{E}_D(Q) + \vec{E}_D(-Q)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{(3R)^2}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{R^2}\hat{i}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\cdot\frac{8}{9}(-\hat{i})$$

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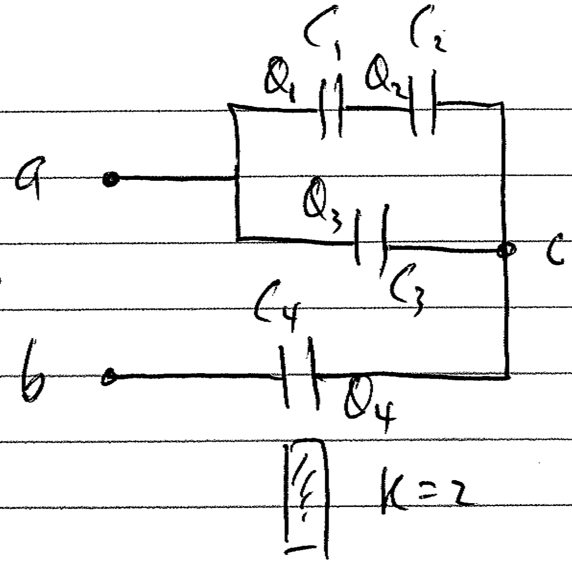
2-(a) We need to know the network capacitance C_{1234} to find the charges on C_4 and then C_3.



$$C_{12} = \frac{C_1 \cdot C_2}{C_1 + C_2} = 2\,nF = 2 \times 10^{-9}\,F$$

$$C_{123} = C_{12} + C_3 = 6\,nF = 6 \times 10^{-9}\,F$$

$$C_{1234} = \frac{C_4 \cdot C_{123}}{C_4 + C_{123}} = \frac{(4\,nF)(6\,nF)}{4\,nF + 6\,nF}$$

$$= 2.4\,nF$$

$$Q_4 = V_{ab} \cdot C_{1234} = 72\,nC = 72 \times 10^{-9}\,C$$

$$Q_4 = Q_{123} = 72 \times 10^{-9}\,C$$

$$\therefore V_{ac} = \frac{Q_{123}}{C_{123}} = \frac{72\,nC}{6\,nF} = 12\,V$$

$\therefore$ Q_3 (charge on C_3) $= C_3 V_{ac} = 48\,nC$ #

Alternatively,

$$Q_3 = C_3 \cdot V_{ac} = C_3 \cdot \left(V_{ab} - V_{cb}\right)$$

$$= C_3 \cdot \left(V_{ab} - \frac{Q_4}{C_4}\right)$$

$$= (4nF)\left(30V - \frac{72nC}{4nF}\right)$$

$$= (4nF)(30V - 18V)$$

$$= (4nF) \cdot (12V)$$

$$= 48nC = 48 \times 10^{-9} C \ \#$$

2-(b) With the dielectric material $k=3$ inserted in C_4, we have $C_4 = kC = 12nF$. Since

$$C_{123} = 6nF$$

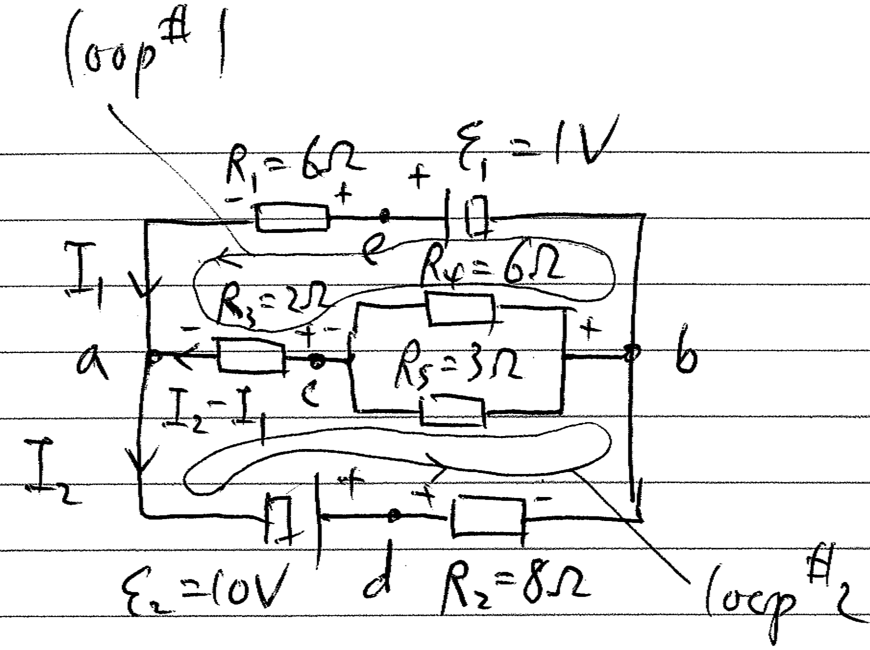
$$\cancel{Q_{1234} = C_{123} + C_4 = 18pF}. \quad C_{1234} = \frac{C_{123} \cdot C_4}{C_{123} + C_4}$$

$$\cancel{Q_4} = \frac{(6nF) \cdot (12nF)}{6nF + 12nF} = 4nF$$

$$\therefore Q_4 = V_{ab} \cdot C_{1234} = 120nC = 120 \times 10^{-9} C \ \#$$

3-(a) $R_{45} = 2\Omega$

$R_{345} = 4\Omega$



Along Loop # 1 :

$$\oint_{a\to c\to b\to e\to a} \vec{E}\cdot d\vec{l}$$

$$= 0 = -(I_2 - I_1) R_{345} - \mathcal{E}_1 + R_1 \cdot I_1$$

$$\therefore \boxed{10 I_1 - 4 I_2 = 1} \quad \text{--- (1)}$$

Loop # 2 :

$$\oint_{a\to d\to b\to a} \vec{E}\cdot d\vec{l} = 0 = -\mathcal{E}_2 + I_2 \cdot R_2 + (I_2 - I_1) R_{345} = 0$$

$$\therefore \boxed{-4 I_1 + 12 I_2 = 10} \quad \text{--- (2)}$$

(1) × 3 + (2) :

$$26 I_1 = 13 \Rightarrow I_1 = 0.5A$$

$$\Rightarrow I_2 = 1A \text{ (through } 8\Omega)$$

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3-(b) $V_{ab} = V_a - V_b = -(I_2 - I_1) \cdot R_{345}$

$= -(1A - 0.5A) \cdot (4\Omega)$

$= -2V$

3-(c) $V_{bc} = (I_2 - I_1) \cdot R_{45} = 1V$

$P_{3\Omega} = P_{R_5} = \dfrac{V_{bc}^2}{R_5} = \dfrac{(1V)^2}{(3\Omega)} = \dfrac{1}{3}$ Watt

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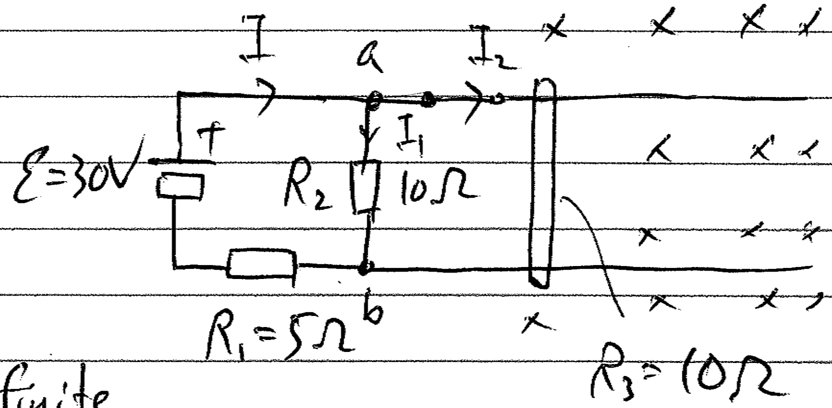
4-(a) Immediately after the switch S is closed, the bar is still not yet moving with a finite velocity, there is no motion emf across the bar.
As a result, the bar is just a resistor with $R_3 = 10\Omega$.



$$R_{23} = \frac{R_2 \cdot R_3}{R_2 + R_3} = 5\Omega$$

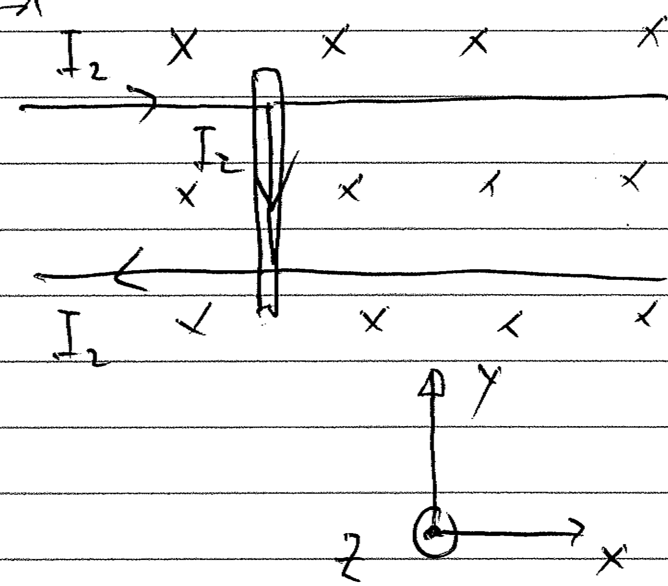
The current through R_1 and R_{23} is

$$I = \frac{\mathcal{E}}{R_1 + R_{23}} = 3A.$$

Since $R_2 = R_3$, $I_1 = I_2 = \frac{I}{2} = 1.5A$.

$\therefore$ The current through the bar is $1.5A$, downward.

4 - (b) Due to the downward current I_2 immediately after the switch is closed, the bar experiences a magnetic force



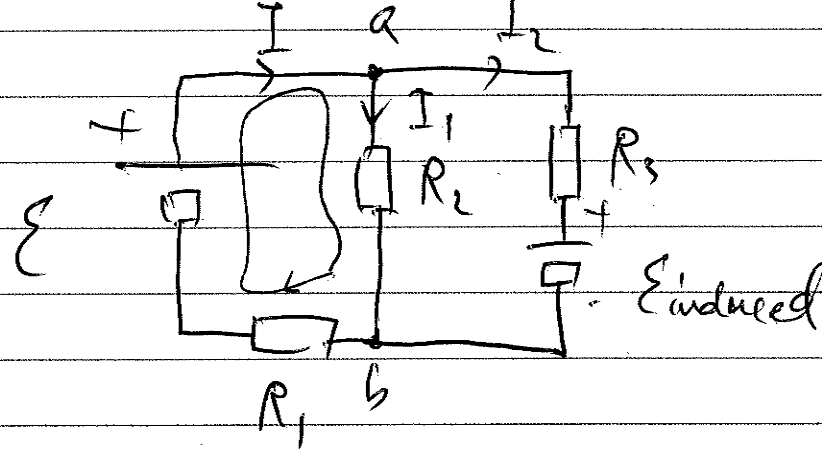
$$\vec{F}^{(M)} = I\vec{l} \times \vec{B}$$

$$= I_2 \cdot l \cdot (-\hat{j}) \times \left(B(-\hat{k})\right)$$

$$= I_2 \cdot l \cdot B \cdot \hat{i}$$

pointing to the left (positive x-direction)

4-(c) As a result of a downward current through the bar, the bar experiences a magnetic force to the left that accelerates the bar. The bar gains velocity to the left. Now the moving bar with the leftward velocity v induces a motion emf $\mathcal{E}_{induced} = B \cdot l \cdot v$



When the induced emf $\mathcal{E}_{induced}$ becomes equal to $V_{ab} = V_a - V_b$ there will be no potential drop across R_3, thus no more current flows through R_3, and in turn the leftward magnetic force goes to zero. As a result, the velocity of the bar does not change anymore. (terminal velocity).

In this case, $I = I_1$, so

$$I = \frac{\mathcal{E}}{R_1 + R_2} = \frac{30V}{5\Omega + 10\Omega} = 2A$$

4-(d) $V_{as} = I_1 \cdot R = I \cdot R = B \cdot l \cdot V_{terminal}$

$$\therefore V_{terminal} = \frac{I \cdot R}{B \cdot l} = 10\ m/s$$

5-(a) The inner current loop produces a magnetic field near the center pointing into the paper, with a magnitude

$$|\vec{B}_{inner\ loop}(center)| = \frac{\mu_0 I_{inner}}{2 \cdot R_{inner}}$$

$$= \frac{4\pi \times 10^{-7}\, T/m \cdot (2A)}{2 \times (0.025\, m)}$$

$$= 5 \times 10^{-5}\, T \quad (0.5\ gauss)$$

To cancel this magnetic field by a field produced by the outer loop with a radius $R_{outer} = 0.1\, m$, the current I_{outer} must be counterclockwise and the magnitude should be

$$|\vec{B}_{inner}(center)| = |\vec{B}_{outer}(center)|$$

$$\therefore \frac{\mu_0 I_{inner}}{2 R_{inner}} = \frac{\mu_0 I_{outer}}{2 R_{outer}}$$

$$\therefore I_{outer} = I_{inner} \frac{(2 R_{outer})}{(2 R_{inner})} = 8A$$

5-(b) $\vec{m}_{inner} = I_{inner} \cdot \pi R_{inner}^2 \cdot \hat{n}$ with $\hat{n}$ pointing into the paper.

$$|\vec{m}_{inner}| = (2A) \cdot \pi \cdot (0.025m)^2$$

$$= 3.93 \times 10^{-3} A \cdot m^2$$

The maximum torque by the outer loop is that when the inner loop is perpendicular to the outer loop

$$|\vec{\tau}_{on\ inner\ loop,\ Max}|$$

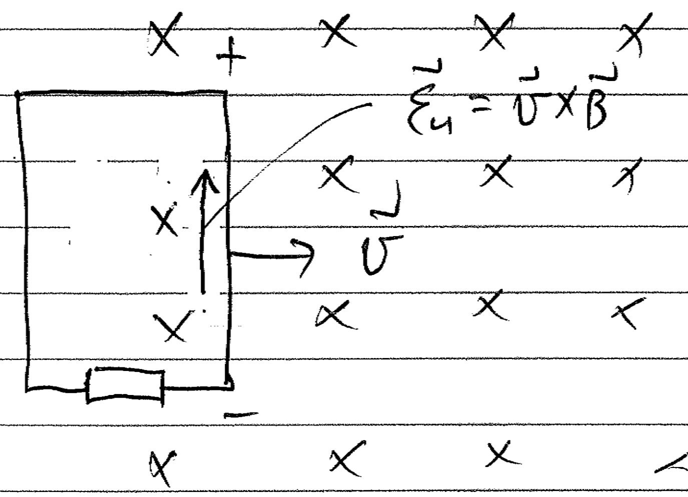
$$= |\vec{m}_{inner}| \cdot |\vec{B}_{outer}(center)|$$

$$= (3.93 \times 10^{-3} A \cdot m^2) \cdot \frac{4\pi \times 10^{-7} T/m \cdot (4A)}{2R_{outer}}$$

$$= 9.9 \times 10^{-8} N \cdot m$$

6-(a) As the wire loop enters so that it is partly inside, the right segment induces a motion emf $\mathcal{E}_{induced} = B \cdot l \cdot v$

This causes a counterclockwise current

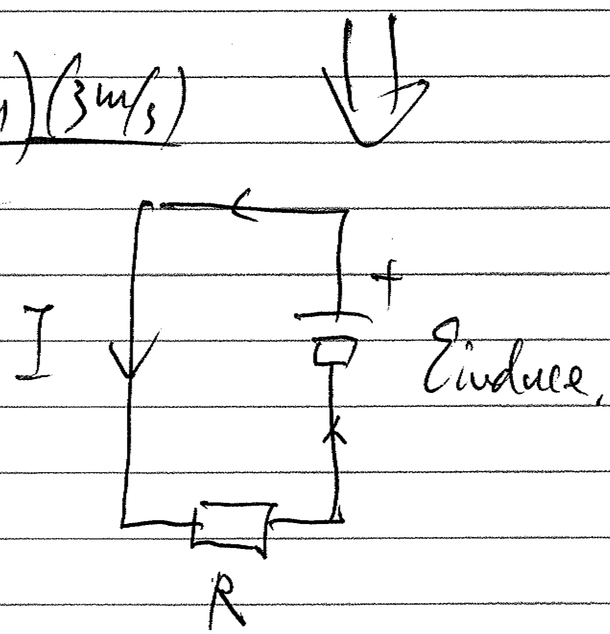


$$I = \frac{\mathcal{E}_{induce}}{R}$$

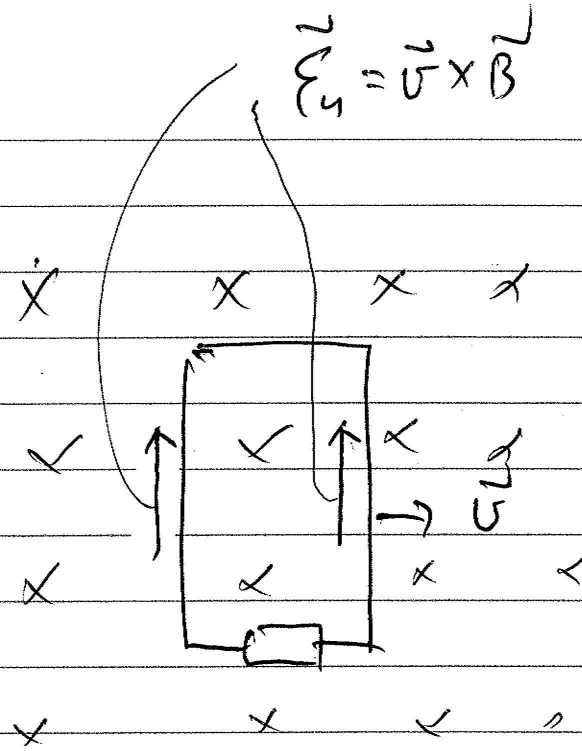
$$= \frac{B \cdot l \cdot v}{R}$$

$$= \frac{(1.25T)(0.75m)(3m/s)}{1.25\Omega}$$

$$= 2.25A$$

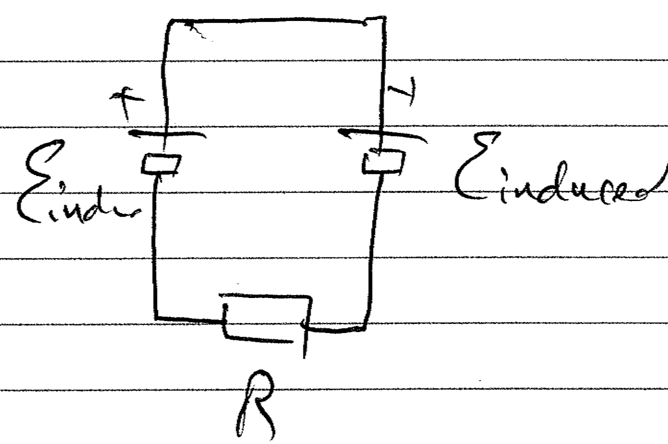


6- (b) As the entire loop is inside the magnetic field, both the right and left segments induce a motion emf

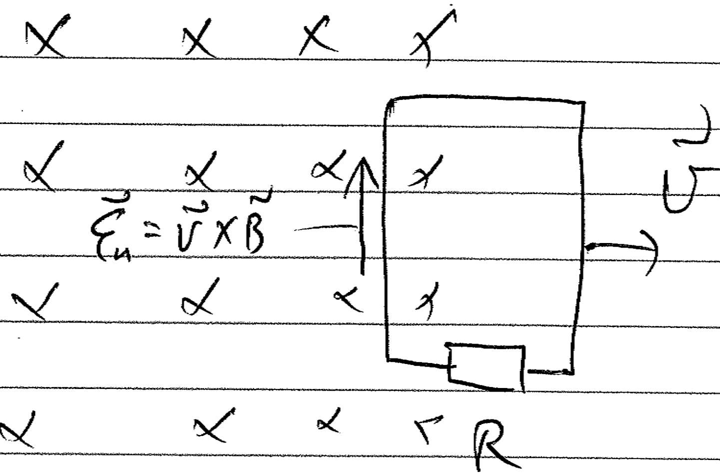


$\mathcal{E}_{induce} = B \cdot l \cdot v$

Since they are equal, and oriented opposite to each, they produces no net current in the loop.



6-(c) As the loop is moving out of the magnetic field while still partly inside, the left segment induces a motion emf $\mathcal{E}_{induced} = B\,l\,v$

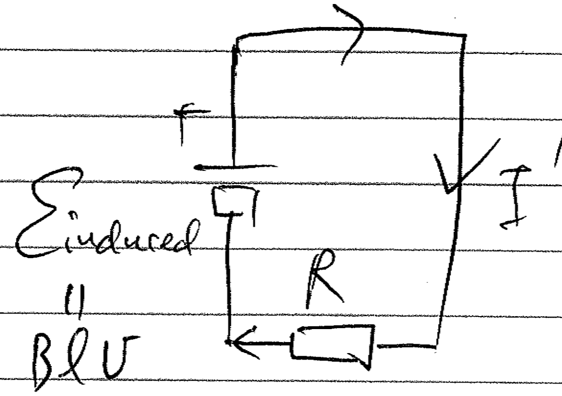


This causes a clockwise current

$$I' = \frac{\mathcal{E}_{induced}}{R}$$

$$= \frac{(1.25T)(0.75m)(3m/s)}{(1.25\Omega)}$$

$$= 2.25\,A$$



7-(a) The total flux change

$$|\Delta \Phi_B| = B \cdot \pi \cdot R^2$$

$$= B \cdot \frac{\pi}{4} d^2$$

$$= (1.5\,T) \cdot \frac{\pi}{4} \cdot (3 \times 10^{-2}\,m)^2$$

$$= 1.06 \times 10^{-3}\,T \cdot m^2$$

Over a time of $\Delta t = 0.25$ sec, the average rate of the flux change through the loop is

$$\left| \left\langle \frac{d\Phi_B}{dt} \right\rangle \right| = \frac{|\Delta \Phi_B|}{\Delta t} = 4.24 \times 10^{-3}\,T \cdot m^2/s$$

This equals to the averaged induced emf in the loop.

$$\langle \mathcal{E}_{induced} \rangle = \left| \frac{d\Phi_B}{dt} \right| = 4.24 \times 10^{-3}\,\text{Volts}.$$

7-(b) The direction of the induced emf is such that it would produce a current to counter the reduction of the magnetic flux into the paper. Thus the current should be clockwise flowing downward through R from a to b.

7-(c) The instantaneous current through the resistor is

$$i(t) = \frac{dq}{dt} = \frac{\mathcal{E}_{induced}}{R} = \frac{1}{R}\frac{d\Phi_B}{dt}$$

$$\int_0^{t_\infty}\left(\frac{dq}{dt}\right)dt = \frac{1}{R}\int_0^{t_\infty}\left(\frac{d\Phi_B}{dt}\right)\cdot dt$$

$$\therefore\ \Delta q = \frac{1}{R}\Delta\Phi_B = \frac{1}{1\Omega} \times 1.06\times10^{-3}\,\mathrm{T\cdot m^2}$$

$$= 1.06\times10^{-3}\,\mathrm{C}$$

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